Vapour properties, cycles and plant

Before embarking on the material set out in this Chapter, you should first make sure you are familiar with the subject matter given in Chapters 12 to 15 of the accompanying Engineering Science book. In particular: when we first look at the nature and behaviour of vapours you will need prior knowledge and understanding of fluid *intensive* properties such as internal energy (U), volume (V), absolute pressure (P) and temperature (T), enthalpy (H) and entropy (S), as well as being familiar with the *extensive or specific* properties of working fluids including; specific volume ((V_s) , specific internal energy (u) , specific enthalpy (h) and specific entropy (s) . In addition, when considering vapour properties you will need to understand latent heat, as detailed in Chapter 12.3.3; fluid processes such as constant volume, constant pressure, isothermal, isentropic and polytropic (Chapter 14.2) and; be able to sketch and interpret pressure/volume and temperature/entropy diagrams as detailed in (Chapter 15.1 and 15.2).

In the book we concentrated on fluids in the gaseous state and to a lesser extent in the liquid state and looked at heat engine applications such as the piston engine and gas turbine engine, where the working fluid was essentially a *gas* or *gas mixture*. Here, we are going to deal with fluids which normally exist in the ***vapour*** phase (a vapour is a fluid in a gaseous state, that *does not* generally obey the gas laws) such as; steam, ammonia, carbon dioxide and Freon and see how some of these fluids may be used in steam power plant and refrigeration systems.

1. Vapour properties

1.1 Introduction

Consider the $p - V$ diagram, Figure 1, that shows the three main states through which the fluid passes, these are:

- *Liquid state*
- *The wet vapour state*, where the fluid consists of a mixture of a liquid and dry vapour and
- *The superheated state* where no liquid particles are present.

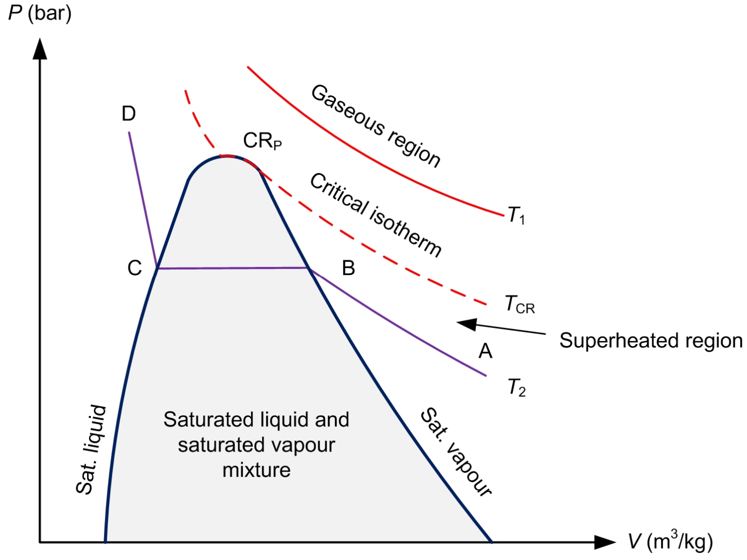


Figure 1 *A fluid undergoing changes of state*

A *saturation state* is defined as *a state at which a change of phase takes place without change in temperature and pressure*. The *saturated liquid line* represents a series of fluid boiling points, joined up at particular temperatures and pressures. Similarly, *the saturated vapour line* represents a series of saturation points where the *liquid* is completely changed into a *vapour*, again without change in their particular pressures and temperatures, as the change of state takes place.

If the fluid is in the *superheated* state (see definition) at a temperature T_2 (point A in Figure 1) and is compressed at this constant temperature along its isotherm (line of constant temperature), it will become a *dry saturated vapour* at point B and then change state at constant pressure between B and C. It becomes a saturated liquid at point C and any further attempt to reduce the volume will cause a rapid rise in pressure, shown as line C to D.

As the temperatures of the isotherms rise towards the critical isotherm T_{CR}, the period of the change of state becomes less until at the *critical point* (CR_P) further compression along this isotherm will produce a change from vapour to liquid without a condensation period. This can clearly be seen by the *reduction in length* of the shaded area (change in state) of the plot in Figure 1, as the isotherms approach T_{CR}, where at the critical point (CR_P) the saturated liquid and dry saturated vapour are physically indistinguishable. At temperatures above the critical isotherm the fluid enters the gaseous state and behaves as a gas, where its properties may be determined using the ideal gas equations, below this isotherm the fluid behaves as a vapour and its properties are then obtained from tables or graphs, as you will see in section 1.2.

To finish our introduction, some common terms associated with vapours and steam, are detailed below.

- *Boiling* – is the formation of vapour bubbles within the fluid, boiling occurs when the absolute pressure in the fluid reaches the vapour pressure.
- *Saturation temperature* – this is the boiling point of the liquid at a given pressure and is the temperature (T_s or t_s) at which the change of state takes place.
- *Saturated vapour* – this is a vapour at the temperature of its boiling point, for a given pressure
- *Dry saturated vapour* – is a vapour in the gaseous state free from liquid particles, all particles being vapourised. Dry saturated vapours *do not* obey the equation of state(characteristic gas equation, see Chapter 12.4.3)
- *Wet saturated vapour* – this is a gaseous mixture (undergoing a change of state) containing saturated liquid and dry saturated vapour, that does not follow the equation of state
- *Dryness fraction* – A fluid (wet vapour) undergoing a change of state between a saturated liquid and a dry saturated vapour, will at any intermediate point have a dryness fraction, given by:

$$\text{Dryness fraction}\,(x) = \frac{\text{mass of dry vapour}}{\text{total mass of wet vapour mixture}}$$

- *Triple point* – This has been chosen as *the datum for water* and is the unique point where *ice, water* and *water vapour* (steam) can co-exist in equilibrium, this point occurs at 0.01°C and 0.006112 *bar* (see reference to tables in Section 1.2)
- *Superheated steam* – this is steam in which the temperature has been raised *above* saturation temperature at a given pressure. For steam in the superheated state temperature and pressure are independent properties and either both of these must be quoted, or as an alternative, the pressure together with the degree of superheat may be quoted, when defining the state of the superheated steam.

1.2 Vapour properties and the use of vapour tables

As mentioned in the introduction, *vapours* (those below the critical temperature) do not obey the perfect gas equations and as such alternative means are often necessary to find their desired properties, one such method requires the use of *vapour tables*.

One such set of tables are those arranged by ***G.F.C. Rogers*** **and** ***Y.R. Mayhew*** entitled ***Thermodynamic and Transport Properties of Fluids*** **(SI Units) Fifth Edition**, published by Blackwell (ISBN 978-0-631-19703-4) that set out, among others, the properties of saturated water and steam, superheated steam and three particular refrigerants; Ammonia, Dichloroflouromethane (Refrigerant 12) and Tetraflouroethane (Refrigerant 134a) . ***You are strongly recommended to obtain a set of these tables or similar, for yourself, before continuing with your study, in order to make sense of their use, in this chapter.*** Other sources of reference, freely available on the net include, for steam: http://www.engineeringtoolbox.com, where tables for saturated steam and the Mollier (enthalpy – entropy) diagram for water-steam (see later) may be found. Also, with respect to refrigerants, information on thermodynamic properties for the environmentally friendly refrigerants such as R – 410A and R – 507 may be found by following the links from the DuPont site at, http://www2.dupont.com/Refrigerants/en_GB/index.html , when consulting the tables presented on this site you should give due consideration to the SI units used where for example, the pressures are given in kPa rather than *bar*.

In this chapter we will only be quoting examples from the tables by *Rogers and Mayhew*, but before we do so we need to define the important properties of vapours and liquids found as headings within these tables.

- ***Specific enthalpy*** (see Chapter 13.4.3) – is defined as the sum of the internal specific energy (u) and the pressure/specific volume energy (pv) i.e. $h = u + pv$, with normal units of kJ/kg .

Important notes:

1) Our usual symbol for *specific volume* V_s used throughout the book has been replaced by (v) since under these circumstances (i.e. its use in thermodynamic tables) there is little possibility of confusing this symbol with that for velocity.

2) From the definition $h = u + pv$, then differentiating $dh = du + pdv + vdp$ and since for the vapour during change of state pressure is constant $dp = 0$, therefore $vdp = 0$ then for an elementary change in heat energy $dQ = du + pdv$, therefore:

$dh = dQ$ ……………………………………………………………………………....1

That is the *change in enthalpy = heat energy change* (heat transferred), compare Equation 1 with Equation 13.11, in the book.

3) The ***subscripts***, used with the properties given in the *Roger and Mayhew tables* and many others, are as follows:

c	*Refers, to a property in the critical state*
f	*Refers, to a property of the saturated liquid*
g	*Refers, to a property of the saturated vapour*
fg	*Refers, to a change of phase at constant pressure* (p)
s	*Refers to a saturation temperature or pressure*

4) The specific enthalpies of the *saturated liquid* $\left(h_f\right)$, *saturated vapour* $\left(h_g\right)$ and superheated vapours are tabulated above the datum corresponding to the triple point of water and steam (see page 2 of Rogers and Mayhew). During the *change of state* from saturated liquid to dry saturated vapour the *enthalpy transferred to the fluid* is $\left(h_{fg}\right)$.

5) In the tables prepared by Rogers and Mayhew they use the same symbol (T) for thermodynamic temperature measured in Kelvin (K) as that for temperature measured in centigrade, *we will also use in this chapter upper case* (T) to *indicate temperature measured in* ($^\circ$C).

- ***Specific enthalpy of wet vapour*** –The dryness fraction (x) defined earlier, indicates the fraction of the enthalpy transferred to the fluid $\left(h_{fg}\right)$, then *the specific enthalpy of the wet vapour*, which is the sum of the enthalpy of the liquid plus the enthalpy of the dry vapour, is given by:

  $h = h_f + xh_{fg}$ ……………………………………………………………………..…2

- ***Specific enthalpy of superheated steam*** – This may be determined directly from tables, such as those shown on page 6 and 7 of Rogers and Mayhew.
  For example steam at 4 bar and 300°C is superheated because the saturation temperature at this pressure is from the table on page 6 $T_s = 143.6$ °C, so the steam at this pressure and temperature has 300 – 143.6 $= 156.4$°C of superheat and has a *specific enthalpy* $h = 3067 kJ/kg$.

- ***Specific internal energy*** – This may be obtained by:

  a) Transposing the definition for specific enthalpy i.e. $h = u + pv$ so that, $u = h - pv$ or

  b) Read directly from tables. Rogers and Mayhew tabulate the specific internal energy of the *saturated liquid* (u_f) and the *dry saturated vapour* (u_g), as well as (u) *for superheated vapour*. When the vapour has *dryness fraction* (x), then the specific internal energy is found from:

$u = u_f + x(u_g - u_f)$ 3

Note from Equation 3 *that the dryness fraction only applies to the change of state*, as it does in Equation 2, when finding the specific enthalpy. By way of example if we wish to find the specific internal energy of wet steam (wet vapour) at 15 bar, with a dryness fraction of 0.8, then from page 4 of the Rogers and Mayhew tables we read-off that, $u_f = 843 kJ/kg$ and $u_g = 2595 kJ/kg$, then from Equation 3 we have $u = 843 + 0.8(2595 - 843) = 2244.6 kJ/kg$

- ***Specific volume*** – This is the volume (v) occupied by unit mass and has units of m^3/kg. Now for *dry vapour* $v = v_g$, this may be found by reading directly from the tables. For *wet vapour* $v = xv_g + (1-x)v_f$ and if the volume occupied by the liquid particles (v_f) is neglected (which in practice it usually is, unless instructed to the contrary) then:

$v \approx xv_g$ 4

Then using as an example our wet steam at 15 bar with dryness fraction of 0.8, the specific volume is found from page 4 of the tables to be $v = (0.8)(0.1317) = 0.10536 m^3/kg$ (ignoring the approximation error).

We are now in a position to find the specific volume, specific internal energy and specific enthalpy properties for wet vapours and for superheat situations. We first look at a couple of examples that involve saturated steam and superheated steam, leaving the refrigerants until we have considered entropy in the next section.
The headings found on pages 2 – 5 of Rogers and Mayhew tables for *saturated water and steam*, are shown below.

$\frac{p}{bar}$	$\frac{T_s}{^\circ C}$	$\frac{v_g}{m^3/kg}$	$\frac{u_f}{kJ/kg}$	$\frac{u_g}{kJ/kg}$	$\frac{h_f}{kJ/kg}$	$\frac{h_{fg}}{kJ/kg}$	$\frac{h_g}{kJ/kg}$	$\frac{s_f}{kJ/kg}$	$\frac{s_{fg}}{kJ/kg}$	$\frac{s_g}{kJ/kg}$

Note that we have used lower case (t) for temperature in Celsius, while in the tables upper case (T) is used.

Example 1

Find the specific volume, internal energy and enthalpy for wet steam at $5bar$ with a dryness fraction of 0.9.

Then from Equation 4 and the listed value of $v_g = 0.3748m^3 / kg$ on page 4 of the tables a good estimate of the *specific volume* of the wet steam is given as $v = xv_g = (0.9)(0.3427) = 0.3084m^3 / kg$.
The *specific internal energy* (u) is given by Equation 1.3 as $u = u_f + x(u_g - u_f)$ so again, from the values on page 4 of the tables we get that $u = 639 + 0.9(2562 - 639) = 2369.7kJ / kg$.
Finally the *specific enthalpy* is given by Equation 2, as $h = h_f + xh_{fg} = 640 + 0.9(2109) = 2538.1kJ / kg$ where by definition $h_{fg} = h_g - h_f$ which in this case from the tables, gives $h_{fg} = 2749 - 640 = 2109$, that is the middle column enthalpy value shown.

Example 2

Steam at a pressure of 100 *bar* has a specific enthalpy of $3097kJ / kg$. Determine the temperature, specific volume of the steam and the specific internal energy.

In order to solve this problem we first need to know whether the steam is *wet* or *superheated.* From page 5 of the tables, wet steam at $100bar$ has a maximum enthalpy in the dry saturated state of $h_g = 2725kJ / kg$ and since the steam we are concerned with has an enthalpy of $3097kJ / kg$ it must be *superheated*. Then from page 8 of the tables we find that superheated steam at 100 bar, with specific enthalpy of $3097kJ / kg$ has a temperature $T = 400^\circ C$ and a specific volume $0.02639m^3 / kg$ (notice that the figures in the tables are 100 times larger than the true values of the specific volume, see the units of specific volume in the column underneath the temperature).

Then to find the *specific internal energy* for superheated steam (not quoted in superheated steam tables, for gaseous state), we may for steam in this condition use our definition of specific enthalpy i.e. $h = u + pv$ so that $u = h - pv = 3097 - (100 \text{ x } 10^2)(0.02639) = 2833.1kJ / kg$. Notice the multiple of 10^2 to convert the pressure in *bar* to kPa and so keep the units for specific enthalpy (kJ / kg) , consistent.

Example 3

Suppose we want to find the temperature and specific enthalpy for say, dry saturated steam at a pressure of $52bar$ then in order to do so we must *interpolate* the values from the tables.

Then from page 5 in the tables the saturation temperature for a pressure of $50bar$ is $T_{50} = 263.9$ °C and at $55bar$ $T_{55} = 269.9$ therefore $T_{52} = 263.9 + \frac{2}{5}(269.9 - 263.9) = 263.9 + 2.4 = 266.3$ °C that is the temperature at $52bar$ is given by adding (in this case) two fifths of the temperature interval to the lower temperature of the interval. If, as a further example, the required temperature had been for a pressure of $53bar$, then the calculation for the interpolation would have been $t_{53} = 263.9 + \frac{3}{5}(269.9 - 263.9) = 263.9 + 3.6 = 267.5$ °C. This kind of interpolation assumes a *linear relationship* and this is the case, verified by the pressure law (see Chapter 12.4.2).

Then in a similar manner the specific enthalpy for the dry saturated steam (h_g) at $52bar$, is given from the table readings as $h_{52} = 2794 + \frac{2}{5}(2790 - 2794) = 2794 - 1.6 = 2792.4 kJ/kg$ noting from the tables that the specific enthalpy *drops* as the pressure rises between 50 and $52bar$.

Note: Extreme care needs to be taken, when interpolating vertically, in the tables, because the linear relationship does not always hold, for example, when considering superheated steam at low pressure (page 6 of tables) the relationship between specific volumes is not at all linear, as can be seen by the graph of pressure against specific volume shown in Figure 1.

1.3 Specific entropy and change of state

Consider Figure 2 that shows the Temperature – Specific entropy diagram for liquid and vapour, where the production of steam at *constant pressure* is represented by line ABCD.

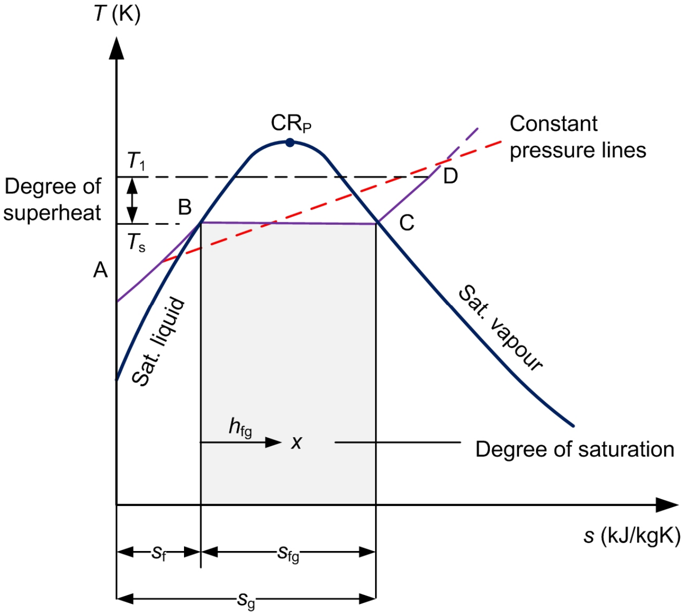


Figure 2 *The $T - s$ diagram for liquid and vapour*

From the diagram it can be seen that the liquid reaches saturation temperature at point B, the change of state takes place between B and C and superheating at constant pressure occurs from C to D. The saturated liquid and dry saturated vapour lines are also shown and the constant pressure line AB can be seen very close to the saturated liquid line. The specific entropies of the saturated liquid s_f, dry saturated vapour s_g and the superheated vapour are all measured back to the triple point, where the entropy is deemed to be zero (see for example the first entry at the top of page 2 in the tables).

The specific entropy of the wet vapour is found in an identical manner to the specific enthalpy and specific internal energy (Equations 2 and 3), where the dryness fraction is only applicable to the change in state and thus the entropy between and including points B, C, is given by:

$$s = s_f + xs_{fg} \quad \ldots\ldots 5$$

Then providing the pressure and temperature of the steam is known and in the case of wet steam the dryness fraction is known or can be determined the entropy (s) can be determined from Equation 5 using the steam tables. Alternatively, providing the specific heat capacity (c_p) is known, the change in specific entropy may be determined using Equation 15.3 from the book i.e. $s_2 - s_1 = \Delta s = c_p \ln\left(\frac{T_2}{T_1}\right) - R\ln\frac{p_2}{p_1}$ and for the circumstance concerning steam at constant pressure, *the change in specific entropy* is given by the relationship:

$$\Delta s = c_p \ln\left(\frac{T}{T_s}\right) \quad \text{.........6}$$

When using Equation 6, remember that *absolute temperature* in Kelvin must be entered.

Note: The shaded area under BC represents the heat transfer during change of state and is equal to the enthalpy, h_{fg} therefore:

$$h_{fg} = T_s\left(s_{fg}\right) \quad \text{.........7}$$

The form of a useful diagram that relates *specific enthalpy* to *specific entropy* is shown in Figure 3, this *enthalpy-entropy diagram* (sometimes known as a *Mollier* diagram) may be used to simplify the calculations involved when dealing with vapours. It is particularly useful when considering the performance of adiabatic steady flow processes, such as flow through diffusers, nozzles and turbines.

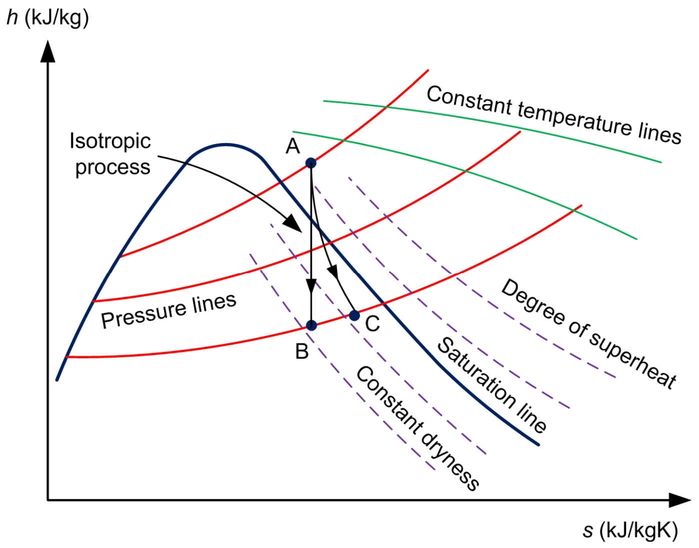


Figure 3 *The h – s diagram for vapour*

If the final conditions of the vapour processes are known then the specific enthalpy or entropy changes may be read directly from the diagram. Thus, if the process is isentropic then this is represented by the vertical line *AB* between the given pressure limits. If the expansion (shown in this case) took place adiabatically (but not reversibly), then this would be represented by the line *AC*, showing the increase in entropy due to friction (see Chapter 15.1.1).

Note: A fully functional *Enthalpy – Entropy diagram for steam*, prepared by *D.C. Hickson* and *F. R. Taylor*, that accompanies the tables by Rogers and Mayhew, may be purchased from numerous book sellers such as Blackwell Publishing of Oxford or obtained on the net at sites such as Amazon, if so desired. A copy of the diagram may also be viewed at: http://www.engineeringtoolbox.com/mollier-diagram-water-d_308.html .The purchase of this diagram is not a requirement for the understanding of the subject matter presented here.

Now that we know a little more about how entropy change is displayed in the tables, we will finish this section with one or two more examples that require the use of steam and *refrigerant* tables.

Example 4

Determine the specific entropy for the dry saturated steam at a pressure of $52bar$ given in Example 3.

Then in a similar manner to the interpolation we used to find the specific enthalpy in Example 1.3, the *specific entropy* for the dry saturated steam (s_g) at $52bar$, is given from the readings on page 5 of the tables as:

$s_{52} = 5973 + \frac{2}{5}(5931 - 5973) = 5973 - 16.8 = 5956.2kJ/kgK$, noting from the tables that the specific entropy (like the specific enthalpy in Example 3) *drops* as the pressure of the dry saturated steam rises between 50 and $52bar$.

Example 5

Refrigerant R12 (dichlorodifluoromethane) is at a temperature $t = 15\,^{\circ}\text{C}$ when the saturation pressure, $p_s = 1.826bar$, find the specific enthalpy and specific entropy of the refrigerant under these conditions.

Then from page 14 of the tables we see that the saturation temperature of the refrigerant at $p_s = 1.826bar$, is $t = -15\,^{\circ}\text{C}$, therefore the refrigerant must have $30K$ of superheat, where the difference in temperature in Celsius between $t = -15\,^{\circ}\text{C}$ and $t = +15\,^{\circ}\text{C}$ is $30^{\circ}\text{C} = 30K$. Then reading off from the 30K superheat column we find that $h = 199.44kJ/kg$ and $s = 0.7728kJ/kgK$.

More practice on the use of vapour tables will be given later when we consider vapour compression cycles in section 3.

2. The Rankine Cycle and Steam Plant

In this section we consider the first of two important vapour cycles, the ***Rankine cycle*** which is the ideal cycle for ***steam plant***. In section 3 we will consider the *Vapour-compression cycle*, for *refrigeration plant*. In order to refresh your memory concerning thermodynamic cycles and thermal efficiencies you should look again particularly at, the Carnot cycle and its associated efficiencies in Chapter 15.2 of the book, before tackling the vapour cycles that follow.

2.1 The basic Rankine cycle

The Rankine cycle as already mentioned is the ideal cycle upon which practical steam plant is based. In its basic form steam power plant consists of a steam generator (boiler), a turbine (the power output from the plant), a condenser and water feed pump, as illustrated in Figure 4.

The steam generated by the boiler at state 1 passes through the turbine where the heat energy from the steam is extracted to do external work (W_T – the whole purpose of the steam plant). The turbine work (from the SFEE that is Equation 13.10 for isentropic expansion) is equal to the enthalpy change across the turbine i.e.

$$W_T = h_1 - h_2 \quad \text{...} 8$$

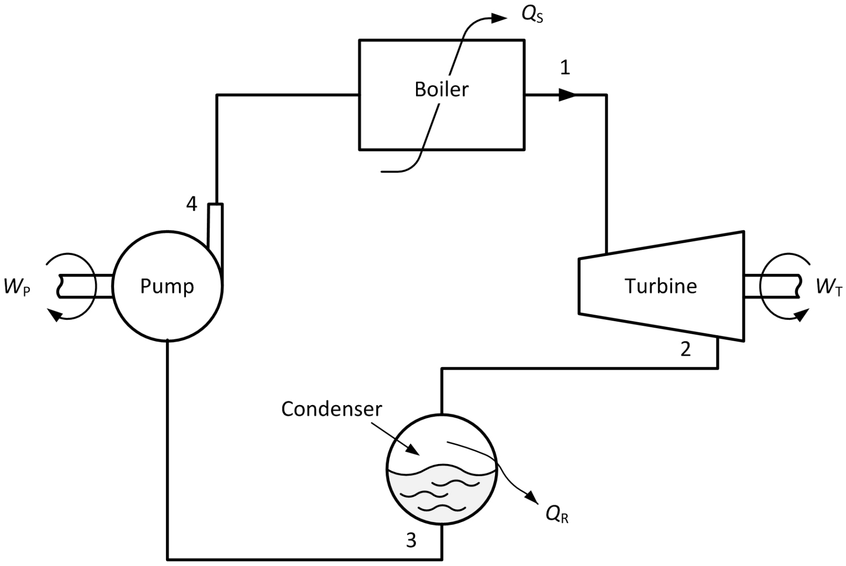


Figure 4 *A typical basic steam plant set-up*

The steam enters the condenser after isentropically expanding through the turbine at state 2. At this stage we make the assumption that only the latent enthalpy (heat) is extracted from the steam, by the cooling water passing through the condenser matrix and that after condensation the water from the steam (now at state 3) enters the feed pump with liquid enthalpy $h_3 = h_{f2}$. Work is then done by the pump in raising the pressure to (p_1) and the condition of the liquid is represented by state 4. The *basic cycle* of the vapour plant shown in Figure 4, is represented on the $p - v$ and $T - s$ diagrams, shown in Figure 5a) and 5b), respectively.

The feed pump work on the liquid is given by, $W_P = (p_1 - p_2)v_{f2}$ (remembering that in this chapter the symbol $v =$ *specific volume*). Heat is transferred (supplied) to the fluid at constant pressure between 4 and 1 (see $p - v$ diagram in Figure 5), therefore, $Q_{in} = h_1 - h_4 = h_1 - \left[h_3 + (p_1 - p_2)v_{f2}\right]$ and the:

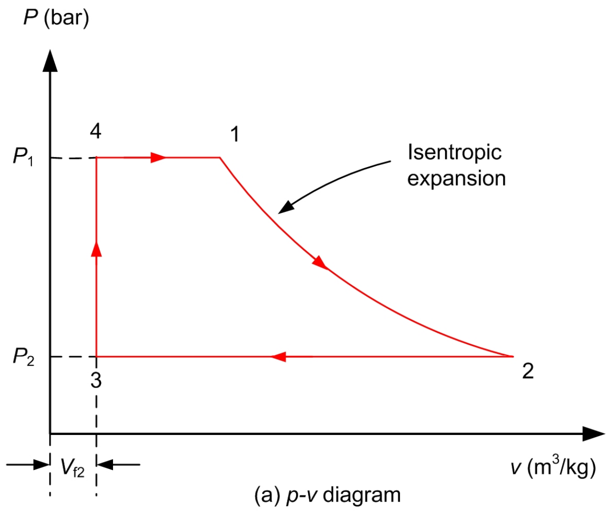
Net specific work done $= W_T - W_P = \left(h_1 - h_2\right) - (p_1 - p_2)v_{f2}$ 9

$$\text{Therefore Rankine efficiency}\left(\eta_R\right) = \frac{\text{net work output}}{\text{positive heat supplied}} \quad \ldots\ldots 10$$

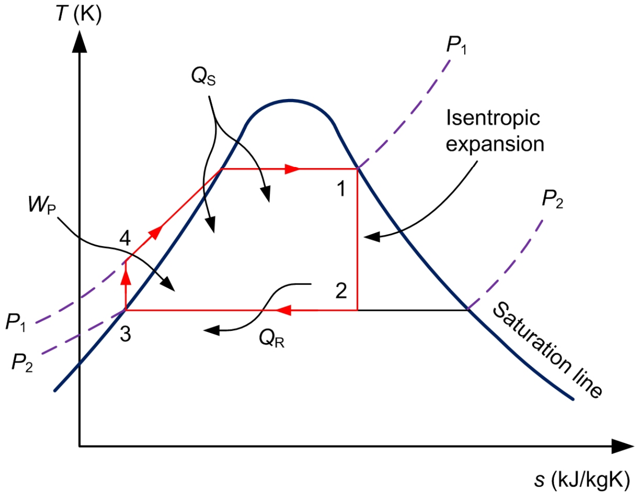
$$\text{Then } \eta_R = \frac{\left(h_1 - h_2\right) - (p_1 - p_2)v_{f2}}{h_1 - \left[h_3 + (p_1 - p_2)v_{f2}\right]} \quad \ldots\ldots 11$$

If the feed pump work $W_P = (p_1 - p_2)v_{f2}$ is ignored, for being small, as it often is then Equation 1.11 reduces to, $\eta_R = \dfrac{h_1 - h_2}{h_1 - h_3}$ and because $h_3 = h_{f2}$ = the specific enthalpy of the liquid at pressure (p_2) then we may write that:

$$\eta_R = \frac{h_1 - h_2}{h_1 - h_{f2}} \quad \ldots\ldots 12$$



(a) *p-v* diagram



(b) *T-s* diagram

Figure 5 *Basic vapour cycle: a) $p-v$ diagram and b) $T-s$ diagram*

Example 6

A steam power plant operating on the ideal Rankine cycle is supplied with steam from the boiler at a pressure of $40bar$ and a temperature of 350°C. The steam exits the turbine and passes to the condenser at a pressure of $0.5bar$. Determine, with the use of tables; a) the dryness fraction of the steam as it enters the condenser, b) the Rankine efficiency, ignoring the feed pump work and c) Sketch the cycle on a $T-s$ diagram. Also d), if the *specific steam consumption* (ssc) is the steam flow in kg/h (kilograms per hour) required to develop 1 kW of power, determine the (ssc) in kg/kWh for this plant.

a) In the ideal Rankine cycle the expansion across the turbine is assumed to be isentropic (see Chapter 14.2.4) therefore there is no change, in entropy so that $s_1 = s_2$. Then from page 4 of the tables (Rogers and Mayhew) the saturation temperature for steam at $40bar$ is 250.3°C, so that in our case the steam supplied to the system at 350°C is *superheated* and from page 7 of the tables steam at $40bar$ at this temperature has entropy $s_1 = 6.584kJ/kgK$. Also from page 2 of the tables, after interpolation the entropy of the steam exhausting the condenser is given (from Equation 5) as, $s_2 = s_f + xs_{fg} = 1.089 + x6.504$ and since $s_1 = s_2$ then $6.584 = 1.089 + x6.504$ from which we get the *dryness fraction* $x = 0.845$

b) Now as we are allowed to ignore the feed pump work, the *Rankine efficiency* is given by Equation 12 $\eta_R = \frac{h_1 - h_2}{h_1 - h_{f2}}$ Where again from the steam tables at $40bar$ and 350°C we find that $h_1 = 3094kJ/kg$. Also, at $0.5bar$ from Equation 2 and page 2 of tables we find that

$h_2 = h_{f2} + xh_{fg2} = 340.2 + (0.845)(2305.1) = 2288kJ/kg$, therefore $\eta_R = \frac{3094 - 2288}{3094 - 340.2} = 0.293 = 29.3\%$

c) Figure 6 shows the sketch of the $T-s$ diagram for our situation, where you should note that the temperature is given in Kelvin, as it should be on thermodynamic diagrams. Note also how the superheated steam entering the plant is illustrated and the isentropic expansion through the turbine is shown by the vertical line from state 1 to state 2.

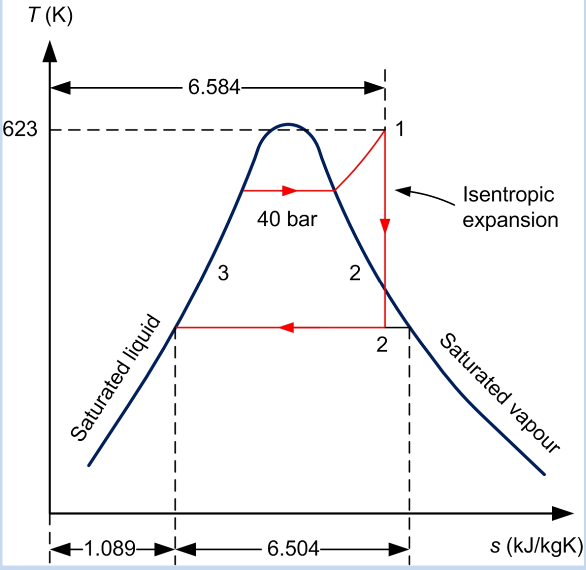


Figure 6 *The $T-s$ diagram for the steam plant operating on the Rankine cycle with superheat*

d) From the information given in the question we know that the *specific steam consumption* is the steam flow in *kg/h* required to develop *1kW*of power. Now $1kWh = (1kJ/s)$ x $(3600s) = 3600kJ$ and the net specific work output, ignoring the feed pump work is, from Equation 1.9, given as,

Net specific work output $= W_T = (h_1 - h_2) = (3094 - 2288) = 806kJ/kg$ and remembering that $1kWh = 3600kJ$,

then the specific steam consumption $ssc = \frac{3600}{806} = 4.47kg/kWh$

2.2 The improved Rankine cycle and steam plant

The energy provided from the turbine of steam plant, is frequently used to produce electrical power where even small increases in thermal efficiency can provide large savings in the fuel requirements of the plant. It is for this reason that every effort is made to increase the efficiency of the Rankine cycle upon which steam plants operate. The methods used to improve the efficiency of the Rankine cycle are based on being able to:

- *Increase the average temperature at which heat is transferred to the working fluid in the boiler or*
- *Decrease the average temperature at which heat is rejected from the working fluid in the condenser*

There are three ways in which this may be accomplished, these are:

i) Superheating the steam to higher temperatures
ii) Lowering the condenser pressure
iii) Increasing the boiler pressure

2.2.1 Superheating

The average temperature at which heat is added to the steam can be increased, without increasing boiler pressure, by superheating the steam. We have previously determined the degree of superheat but have not until now, really given a reason for its use.

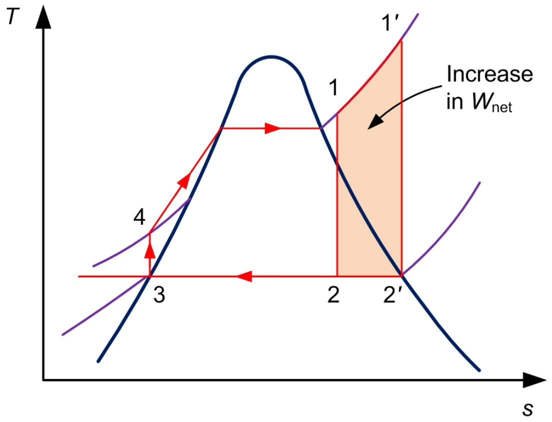


Figure 7 *The effect of superheating*

Figure 7 illustrates the effect of *superheating* (our steam plant set-up Figure 4), where it can be seen that increasing the average temperature at which heat is transferred to the working fluid in the boiler increase the *heat input* and the *network*, resulting in an increase in thermal efficiency.

2.2.2 Lowering condenser pressure

We know from our study of the basic system that, steam exists in the condenser as a saturated mixture, at a saturation temperature directly related to condenser pressure so that lowering the condenser pressure, automatically lowers the temperature of the steam.

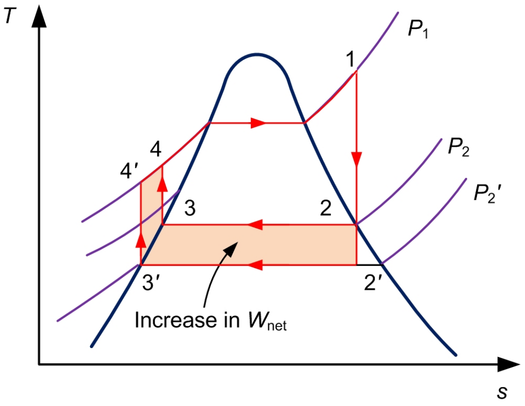


Figure 8 *The effect of lowering the condenser pressure*

From Figure 8 it can be seen that lowering the temperature of the steam increases the network of the plant and so increases its efficiency, this can also be seen by studying the above Equations 10 and 11. A consequence of lowering condenser pressure $(2', 3')$ is that it increases the moisture content of the steam in the final stages of the turbine (higher percentage of saturated vapour at $2'$) which is detrimental to the performance of the turbine causing a reduction in efficiency and over time, corrosion of the turbine blades.

2.2.3 Increasing boiler pressure

An increase in boiler pressure results in an increase in the temperature at which boiling takes place. This in turn raises the temperature at which heat is added to the steam, raising the thermal efficiency of the cycle.

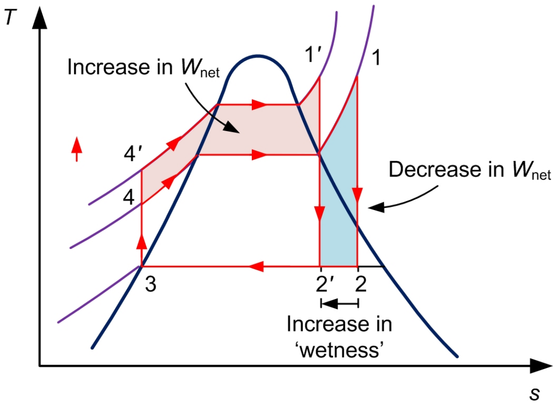


Figure 9 *The effect of increasing boiler pressure*

From Figure 9 it can be seen that for a fixed turbine inlet temperature $(1', 1)$ that the cycle moves to the left, increasing the moisture content of the steam, as it exits the turbine. This undesirable effect can be minimised by superheating the steam to very high temperatures prior to turbine entry or expanding the steam in stages using a process known as *reheating*, this latter process provides the best practical solution to the problem as you will see in section 2.2.4.

Example 7

Steam from the boiler enters the turbine of a steam plant (operating on the Rankine cycle) at $40bar$ and 350°C it exits the turbine and passes to the condenser at a pressure of $0.05bar$. Determine:

a) The thermal efficiency of the power plant
b) The thermal efficiency if the steam is superheated to 600°C, prior to entering the turbine
c) The thermal efficiency if the boiler pressure is raised to $150bar$ while the turbine entry temperature remains at 600°C.

a) Now in order to find the thermal efficiency of the plant we proceed in a similar manner to that adopted for Example 6, where in this case the steam enters the turbine under the same conditions but leaves under different conditions. Then using Equation 5 and page 3 of the tables by Rogers and Mayhew for a pressure of $0.05bar$, we find the dryness fraction from $s_2 = s_f + xs_{fg} = 0.476 + x7.918$ and because we assume isentropic expansion through the turbine for plant based on the Rankine cycle, then $s_1 = s_2 = 6.584kJ/kgK$ so $6.584 = 0.476 + x7.918$ and the dryness fraction $x = 0.7714$, also from page 7 of the tables for superheated steam at 350°C we find that $h_1 = 3094kJ/kg$.

We are now in a position to find the specific enthalpy of the vapour as it leaves the turbine, then from Equation 2 and page 3 of the tables, $h_2 = h_{f2} + xh_{fg2} = 138 + (0.7714 \text{ x } 2423) = 2007.5 kJ / kg$. Also from Equation 9 the pump work is given by $W_P = (p_1 - p_2)v_{f2}$ and since $v = v_f$ at 0.05bar then from page 10 of the tables, v_f □ 0.1005 by interpolation, therefore in consistent units $W_p = 0.001005(40 - 0.05)\left(\frac{10^5}{10^3}\right)$ so that $W_p = 4.01 kJ / kg$. Then from Equation 11 where $\eta_R = \frac{(h_1 - h_2) - (p_1 - p_2)v_{f2}}{h_1 - [h_3 + (p_1 - p_2)v_{f2}]}$, noting that $h_3 = h_f = 138 kJ / kg$ at $0.05 bar$, then we find that $\eta_R = \frac{(3094 - 2007.5) - 4.01}{3094 - [138 + 4.01]} = 36.7\%$

b) Again from page 7 of the tables for the superheated vapour at $40 bar$ and 600°C then $h_1 = 3674 kJ / kg$ and for the Rankine cycle $s_1 = s_2 = 7.368 kJ / kgK$ therefore from $s_2 = s_f + xs_{fg}$, $7.368 = 0.476 + x7.918$ we find that the dryness fraction is $x = 0.87$, from Equation 2 and page 3 of the tables $h_2 = h_{f2} + xh_{fg2} = 138 + (0.87 \text{ x } 2423) = 2246 kJ / kg$ and because $h_3 = h_f$ then $h_3 = 138 kJ / kg$. The feed pump work will remain the same as that found in part a) because the entry and exit pressures in the turbine are the same therefore the efficiency is from Equation 11, found to be, $\eta_R = \frac{(3674 - 2246) - 4.01}{3674 - [138 + 4.01]} = 40.3\%$.

Note: that by increasing the superheat temperature of the steam from 350°C to 600°C we have improved the efficiency, of the plant from 36.7% to 40.3%, while at the same time increasing the dryness of the steam leaving the turbine from 71% to 87%.

c) By increasing the boiler pressure to $150 bar$, there will be changes to the states in the cycle, these are highlighted in the notes below.

From the tables $h_1 = 3581 kJ / kg$, $s_1 = s_2 = 6.677 kJ / kgK$ and $6.677 = 0.476 + x7.918$ so the dryness fraction of the steam leaving the turbine is $x = 0.783$. The enthalpy of the vapour leaving the turbine is given by $h_2 = h_{f2} + xh_{fg2} = 138 + (0.783 \text{ x } 2423) = 2035.6 kJ / kg$. The pump work will now be $W_P = (p_1 - p_2)v_{f2} = (150 - 0.05)0.001005 \text{ x } \left(\frac{10^5}{10^2}\right) = 15.07 kJ / kg$.

The cycle efficiency is now $\eta_R = \frac{(3581 - 2035.6) - 15.07}{3581 - [138 + 15.07]} = 44.6\%$

Notes: By increasing the boiler pressure, from $40 bar$ to $150 bar$ the thermal efficiency is increased from 40.3% to 44.6% however the dryness fraction of the steam leaving the turbine is reduced from 87% down to 78.3%, in other words the steam leaves the boiler in a *wetter* condition.
During this example we have only considered improvements based on the *ideal* Rankine cycle, let us suppose that in part a) the turbine working in the real plant has an isentropic efficiency of 85% what effect will this have on the plant efficiency? We know that the turbine work is given as $W_T = h_1 - h_2$ and the isentropic efficiency $0.85 = \frac{h_1 - h_{2'}}{3094 - 2007.5}$ so that the real turbine work, in this case $h_1 - h_{2'} = 923.5 kJ / kg$. The plant efficiency will now be $\eta_R = \frac{(923.5) - 4.01}{3094 - [138 + 4.01]} = 31.1\%$

2.2.4 Rankine cycle with re-heat

As explained in section 2.2.3, an unfortunate consequence of increasing the efficiency of the plant by increasing the boiler pressure is that the moisture content of the steam is also increased to unacceptable levels as it exits the turbine (see Figure 9). We know from the above argument that superheating the steam prior to entering the turbine would be one solution. Unfortunately the very high temperatures required are prohibited by the temperature tolerance of the materials from which the turbine blades are made, although current and future developments in materials technology, have allowed turbine designers, to increase entry temperatures, all be it at an additional cost.

However the current solution which results in notable efficiency gains is to use the concept of *reheating the steam* in the boiler after passing from the high pressure turbine before reaching the low pressure turbine. In this way the degree of wetness of the steam leaving the low pressure turbine is kept to a suitable value, normally this is around 10%. Example 8 that follows, illustrates the benefits to be gained by the application of reheat to a steam plant that operates on this modified Rankine cycle.

Example 8

Consider a steam plant with reheat, working under the same conditions as in Example 6c), where in this case the steam at $150 bar$ and 600°C enters the *high pressure* turbine and leaves this turbine at a pressure of $0.05 bar$, as before. If the moisture content of the steam at exit from the *low pressure* turbine is limited to a maximum of 10% and assuming that the steam is reheated again to 600°C, find:

a) The reheat pressure of the steam
b) The thermal efficiency of the plant.

Figure 10 illustrates the situation both schematically and on a T – s diagram.

a) We know that the plant is working on the ideal Rankine cycle therefore the pump and turbines are isentropic and there are no pressure losses in the boiler or condenser, as shown on the $T-s$ diagram in Figure 10.

We need to find all the conditions of the steam as it enters and leaves the low pressure turbine, so that in particular, we can find the reheat pressure. To do this we proceed in a similar manner as before working on the fact that the entropies at entry and exit from the LP turbine are the same and a saturated steam mixture leaves the turbine.

We are told that the wetness of the steam must not exceed 10%, as it leaves the LP turbine (point 6) in other words the dryness fraction of the steam must not be less than $x = 0.9$. Therefore, using equation 5 and page 3 of the tables where the exit pressure is 0.05 *bar* we find that:

$$s_6 = s_f + x_6 s_{fg} = 0.476 + (0.9)(7.918) = 7.6022 kJ/kgK .$$

The specific enthalpy of the steam at point 6, as it exits the LP turbine may also be found, knowing the dryness fraction using Equation 2 and page 3 of tables, so:

$$h_6 = h_f + x_6 h_{fg} = 138 + (0.9)(2423) = 2318.7 kJ/kg .$$

Now the specific entropy at point 5 at entry to the LP turbine is the same as that at exit, i.e. $s_5 = s_6 = 7.6022 kJ/kgK$ and since we are given that $T_5 = 600^\circ C$, then from page 7 of the tables and interpolation we find that the *reheat pressure* $p_5 = 24.9 bar$, while the specific enthalpy of the steam at entry to the LP turbine is $h_5 = 3686 kJ/kg$.

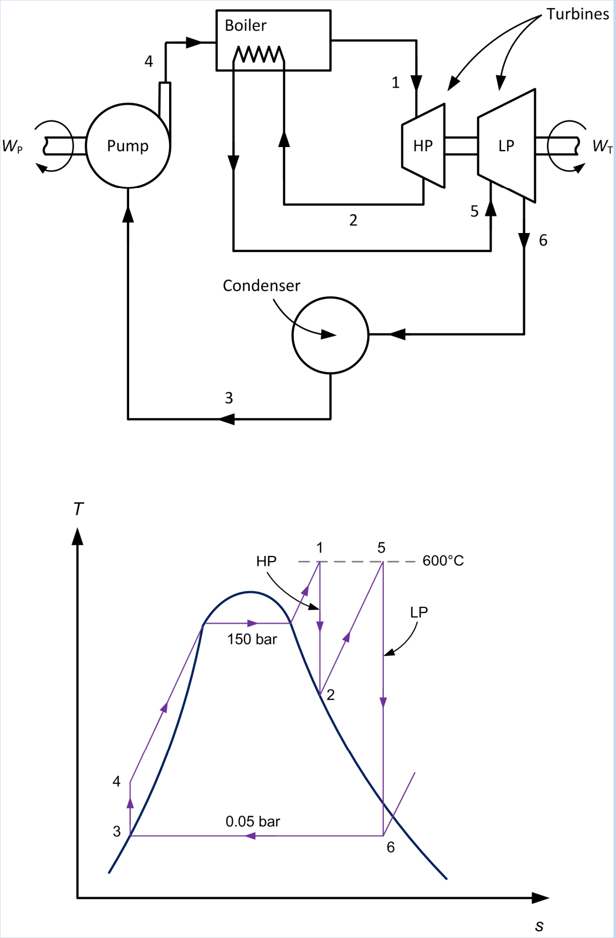


Figure 10 *Schematic and T – s diagram of steam plant with reheat*

b) In order to find the efficiency of the plant we need to find the *specific enthalpies* of the working fluid (steam) *at each point* (state) in the cycle using tables and then, find the work transfers of the pump and turbines, using equations (9 to 11).

Then for *point 1*, where $p_1 = 150 bar$ and $T_1 = 600^\circ C$ we find from page 8 of the tables that, $h_1 = 3581 kJ/kg$ and $s_1 = 6.677 kJ/kgK$.

Point 2, where we know that $p_2 = p_5 = 24.9 bar$ and $s_2 = s_1 = 6.677 kJ/kgK$. If, we now interpolate values from page 7 of the tables we find that our value of the specific entropy lies on the column where $T = 300^\circ C$, at a pressure $p_2 = p_5 = 24.9 bar$, so interpolating for the specific enthalpy between the value 2995 to 3025 kJ/kg K we find that $h_2 \cong 3010 kJ/kg$.

Point 3, where the working fluid is a saturated liquid that enters the pump at a pressure of $0.05 bar$, so the value of its enthalpy $h_3 = h_f$ may be read directly from page 3 of the tables as $h_3 = 138 kJ/kgK$. For this saturated liquid the specific volume is $v_3 = v_f$ which when interpolated from page 10 of the tables for a pressure of 0.05 *bar* is $v_3 = 0.0010053\, m^3/kg$ (note units from table). Now since through the pump we have constant entropy and $p_4 = 150 bar$, then from Equation 9 where in this case $W_P = (p_4 - p_3)v_{f3}$ we find the work input to the pump is:

$$W_P = (p_4 - p_3)v_{f3} = (15000 - 5)(0.0010053) = 15.074 kJ/kg.$$

Now from the $T-s$ diagram in Figure 10, you should be able to deduce that the specific enthalpy at *point 4* will be: $h_4 = h_3 + W_p = (138 + 15.074) = 153.074 kJ/kg$.

Now we know from equation 10, that Rankine efficiency $(\eta_R) = \dfrac{\text{net work output}}{\text{positive heat supplied}} = 1 - \dfrac{q_{out}}{q_{in}}$ and

$q_{in} = (h_1 - h_4) + (h_5 - h_2) = (3581 - 153.074) + (2318.7 - 138) = 4103.926 kJ/kg$ also,

$q_{out} = (h_6 - h_3) = (2318.7 - 138) = 2180.7 kJ/kg$

Therefore the plant thermal efficiency, working on the Rankine cycle is

$$\eta_R = 1 - \frac{2180.7}{4103.926} = 0.469 = 46.9\%$$

The above efficiency compares with that given for the cycle without reheat (example 7) of 44.6% that is a 2.3% increase in efficiency.

We leave this short study of steam and steam plant, if you require more information, for example, on the benefits of plant that include regeneration or combined systems, you are advised to consult standard undergraduate texts on thermodynamics such as those written by G.F.C. Rogers and Y.R. Mayhew or the text on Applied Thermodynamics by T.D. Eastop and A. McConkey or any of their modern equivalents.

We now take a brief look at vapour compression refrigeration and its associated plant.

3. Vapour Compression Refrigeration and Plant

3.1 Refrigerants

The ideal form of the vapour compression cycle is in fact, the *Carnot heat engine cycle* (Chapter 15.2) acting in *reverse* and this is the ideal cycle used for refrigerators and refrigeration plant, where the working fluid is a *refrigerant* capable of evaporation at cold temperatures, so absorbing heat in the form of latent energy. Thus, the refrigerant used passes through a closed cycle during which it absorbs heat from the cold chamber of the refrigeration unit, it is then compressed to an appropriate high pressure and temperature, cooled in a *condenser* and then *throttled down* to the pressure at which *evaporation* can take place, (see Figure 11).

The ***refrigerant*** should be capable of producing a good refrigerating effect, for the amount of work put into the cycle. Some of the important properties of refrigerants that govern their choice for particular applications include; level of toxicity, corrosive/non-corrosive properties, volatility, flammability and with modern day refrigerants, the effect they have on the environment is of importance. Refrigerants are all identified using a refrigerant (R) number. The older typical refrigerants include *ammonia* (R717) and *carbon dioxide* (R744), together with fluorocarbons (CFCs) such as *Freon*, all of these posing some threat to the environment, as well as in the case of ammonia, having high toxicity and flammability.

The tables (*Rogers and Mayhew*) show the properties of Ammonia (R717), Dichlorodifluoromethane (R12) and the inert gas refrigerant, Tetrafluoroethane (R134a), note that in each case on pages 13, 14 and 15 of the tables only two superheat temperatures are given. Thus for example on page 14 of the tables, R12 at 1.509 bar and 10°C must have 30K of superheat since its saturation temperature at this pressure is – 20°C and so its specific enthalpy and entropy in this state, are from the tables $h = 196.89 kJ/kg$ and $s = 0.7764 kJ/kgK$, respectively. Modern more environmentally friendly refrigerants have been produced to replace (R12) and (R134a) such as (R410a) which is a mixture of the improved difluoromethane (R32) and pentafluoroethane (R125), which is now used as a refrigerant for air-conditioning applications.

3.2 The basic vapour-compression refrigeration cycle

Figure 11 illustrates the closed cycle for the refrigerant as it passes around the basic vapour-compression refrigeration system.

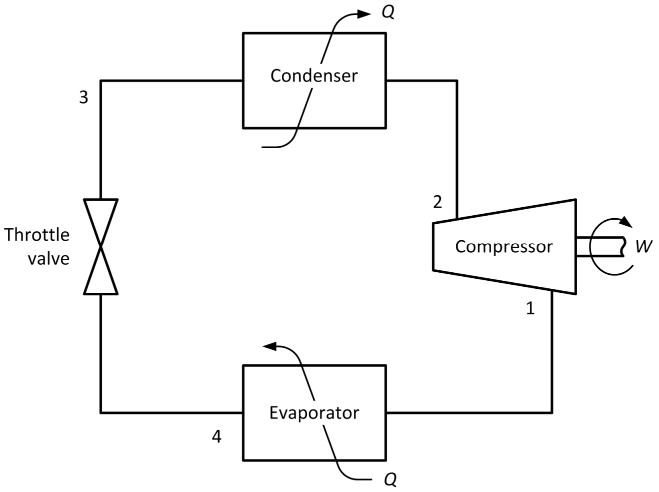


Figure 11 *The basic vapour compression refrigeration cycle*

As the refrigerant passes through the evaporator (which is located in the cold chamber of the refrigerator) heat is absorbed (Q_{in}) and as the refrigerant passes through the condenser (the large coil on the back of a domestic refrigerator), heat is rejected (Q_{out}) the input work (W_{in}) is via the compressor, as shown in Figure 11.

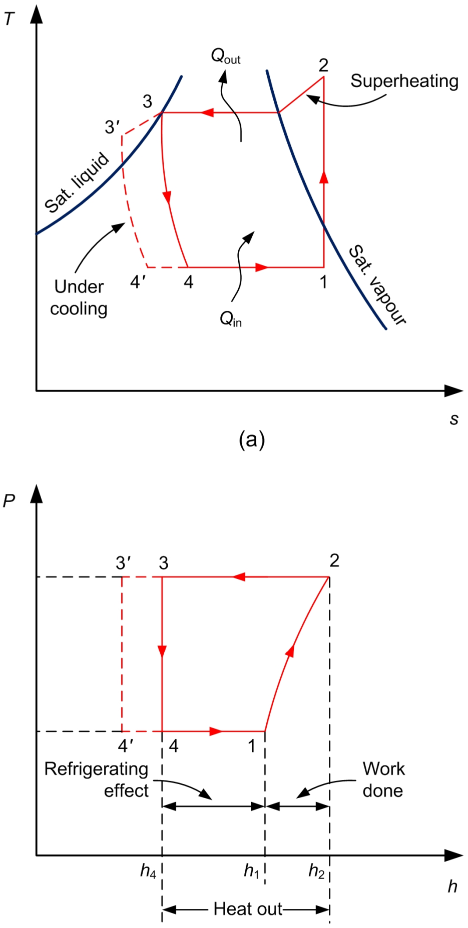
Wet vapour is difficult to expand and compress, so the refrigerant is usually *dry before compression* and *superheated* afterwards. At the exit from the condenser, the under-cooled liquid would be difficult to expand through a turbine so a ***throttling valve*** is used instead. As the refrigerant passes through the throttle, no useful work is produced but this process does convert the pressure into internal energy (with *no change in enthalpy*, see Figure 12b) this makes the liquid evaporate and because its saturation temperature drops, it exits the throttle as a colder very wet vapour (see Figure 12a).

In the vapour compression cycle shown on the $T-s$ and $p-h$ diagrams of Figure 12, we assume that compression is isentropic from 1 to 2 and point 3 is the liquid saturation point at the higher pressure p_2 before throttling. The refrigerant is isentropically compressed from 1 to 2, cooled at constant pressure in the condenser, where in this case, as shown on Figure 12a) it loses its superheat and latent heat. The refrigerant is then throttled to the pressure p_1 where it enters the evaporator as a very wet cool vapour and absorbs heat from the contents of the refrigerator cold chamber, between 4 and 1.

Constant pressure undercooling of the fluid in the condenser between 3 and $3'$, produces the condition at $4'$ after the throttling process has taken place. In the ideal cycle (where we ignore heat losses to the surroundings) the refrigerating effect at the evaporator plus the work done by the compressor on the refrigerant will be equal to the heat given out at the condenser i.e.:
Refrigerating effect + work done on refrigerant = heat out (see Figure 12b).

In practical refrigeration systems, it is found that there is *more* heat energy (Q_{in}) absorbed by the evaporator and more heat energy (Q_{out}) given out at the condenser, than the work needed to drive the compressor (W_{in}).

The ***coefficient of performance*** (C.O.P) is a measure of the refrigerating effect obtained for the work input to the cycle and it is given by the ratio: $COP = \dfrac{\text{Refrigerating effect}}{\text{Work done by the compressor}} = \dfrac{h_1 - h_4}{h_2 - h_1}$



(a)

(b)

Figure 12 $T - s$ and $p - h$ *diagrams for vapour compression refrigeration cycle*

Example 9

Refrigerant 134a enters a throttling valve as a saturated liquid at $7.7bar$ and is throttled to a pressure of $1.0637bar$. Determine the dryness fraction of the refrigerant after throttling and the temperature drop during the process.

This is a simple example of the use of the *Rogers and Mayhew* tables, where the relevant required information is found on page 15. Since the refrigerant is subject to a throttling process the *enthalpy remains constant*. Then at inlet: $p_1 = 7.7bar$ and since refrigerant is a saturated liquid the temperature

$T_1 = T_s = 30^\circ C$ and $h_1 = h_f = 241.69kJ/kg$.

At exit $p_1 = 1.0637bar$, $h_f = 167.25kJ/kg$, $h_g = 383.37kJ/kg$ and $T_s = -25^\circ C$. Now since for throttling process $h_1 = h_2$ and using Equation 2, $h = h_f + xh_{fg}$ we find that the dryness fraction of the refrigerant is $x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{241.69 - 167.25}{383.37 - 167.25} = \frac{74.44}{216.12} = 0.344$. Thus the refrigerant at exit from the throttling valve is a, saturated vapour with a dryness fraction of 0.344, therefore the temperature of the refrigerant at exit must be the saturated temperature at this pressure, i.e. $T_s = -25^\circ C$ and so the temperature change $\Delta T = T_2 - T_1 = -25 - 30 = -55^\circ C$, that is a temperature drop of 55°C.

Example 10

In a vapour compression refrigeration system using R12, evaporation take place at $1.509 bar$ and condensation occurs at $6.516 bar$. The vapour is dry saturated after isentropic compression and there is no undercooling, also the vapour is throttled between leaving the condenser and entering the evaporator. Using appropriate values from the tables sketch the resulting cycle on a $T-s$ diagram and determine:

a) The temperatures and specific enthalpies at points 2, 3 and 4, in the cycle
b) The dryness fraction of the vapour on entry to the condenser and h_1
c) The coefficient of performance of the system

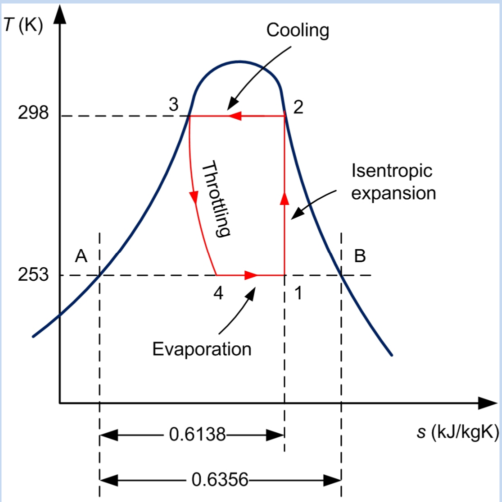


Figure 13 *$T-s$ diagram for R12 vapour compression refrigeration system*

a) The cycle is shown in Figure 13, where it can be seen that reading directly from page 14 of the Rogers and Mayhew tables for R12, $T_1 = -20 + 273 = 253K$ and $T_2 = 25 + 273 = 298K$. Also the appropriate specific enthalpies are as follows $h_2 = h_g = 197.73 kJ/kg$, $h_3 = h_f = 59.7 kJ/kg$ at $298K$ and because the refrigerant is throttled from point 3 to point 4, then $h_4 = h_3 = 59.7 kJ/kg$.

b) The dryness fraction at entry to the condenser may be found by considering the entropy at points A and B, shown on Figure 13 and since for isentropic compression $s_1 = s_2$, then the required dryness fraction $x_1 = \dfrac{A1}{AB} = \dfrac{0.6869 - 0.0731}{0.7087 - 0.0731} = \dfrac{0.6138}{0.6356} = 0.966$, taking the values of specific entropy from page 14 of tables.

The *enthalpy at point 1* may now be found using Equation 2 where $h_1 = h_f + x_1 h_{fg}$ so that, $h_1 = 17.82 + 0.966(178.73 - 17.82) = 173.26 kJ/kg$.

c) The coefficient of performance is now found easily using $COP = \dfrac{h_1 - h_4}{h_2 - h_1} = \dfrac{173.26 - 59.7}{197.73 - 173.11} = 4.61$.

Notice that the coefficient of performance is greater than unity this is allowable and happens when the heat removed from the refrigerating space is greater than the work input. The COP should not be confused with the thermal efficiency which can never be greater than unity!

3.3 Modified vapour cycles and refrigeration plant

As discussed in the previous section wet vapour is difficult to expand and compress, so the refrigerant is usually *dry before compression* and *superheated* afterwards (Figure 12a). At the exit from the condenser, the under-cooled liquid used to increase the refrigerating effect, would be difficult to expand through a turbine, hence the use of a *throttling valve*. Improvements may be made to this basic compression cycle, by use of a ***flash chamber.***

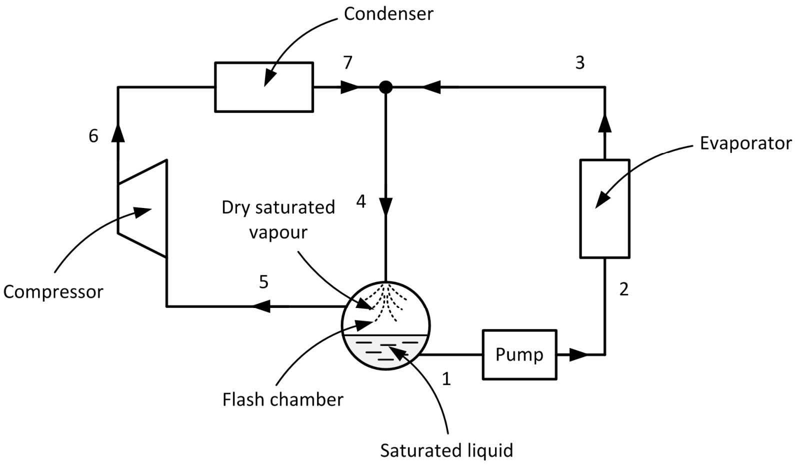


Figure 14 *Schematic of basic flash chamber vapour refrigeration system*

In the system schematic shown in Figure 14, the condensed high pressure liquid at (point 7) is sprayed into the low pressure flash chamber (via point 4) where, the drop in pressure has the same effect as throttling and the liquid partially evaporates and drops in temperature. Now in this set-up the dry saturated vapour is drawn into the compressor (point 5), *rather than* passing directly through the evaporator, where, if it did so, it would make no contribution to the refrigerating effect. In the mean-time the saturated liquid from the flash chamber (point 1) is pumped to the evaporator (2). The advantage of this system is that the evaporator now works at a higher pressure, which in turn increases the refrigerating effect (see Figure 12b).

Further improvements to the flash chamber system may be made if an increase in plant components was acceptable, where efficiency gains out-weighed the increase in the size and costs of the plant.
In the more complex plant illustrated in Figure 15, we still use a flash chamber where the vapour is bled off and fed back to the compression process at some intermediate pressure but in this case the throttling process is carried out in *two stages*, each one starting with a liquid.

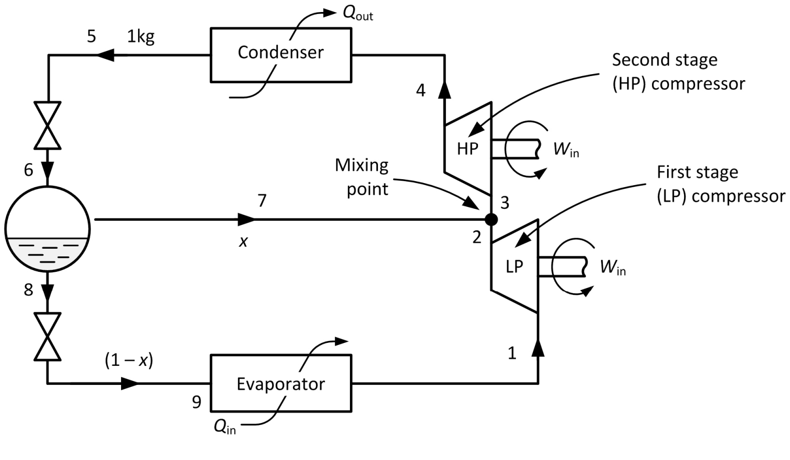


Figure 15 *Schematic of a two-stage flash chamber vapour refrigeration system*

Consider a quantity of refrigerant flowing around the system illustrated in Figure 15. At the flash chamber a quantity of the refrigerant, as a dry saturated vapour is bled-off (7) at some intermediate pressure to the inter-stage of the two stage compressor (3). The remaining liquid in the flash chamber passes through a throttling valve (8) to the evaporator (9), which in turn passes through the first stage of the compressor (1-2) where, as a vapour, it is mixed adiabatically with the flash vapour (7). The resulting mixture at stage (3) is compressed in the second stage compressor (3-4) where it is then delivered to the condenser at state (4). On leaving the condenser (5) the saturated liquid is throttled and at (state 6) is sprayed into the flash chamber.

Now the problem with the analysis of the system illustrated in Figure 15 is that the quantities of the refrigerant circulating around the system *are not the same*, so we need to look at how we determine these quantities and the state of the refrigerant at various points around the system. If we look at unit mass of the fluid flowing around the system, then at point (7) we see from the Figure that x *kg* of dry saturated vapour is bled-off to the inter-stage mixing point, the remaining mass $(1-x)kg$ of liquid refrigerant, passes through the throttle valve (8-9) to the evaporator. Formula 2 i.e. $h = h_f + xh_{fg}$ may be used to determine the values of x and so the energy balance, on the flash chamber.

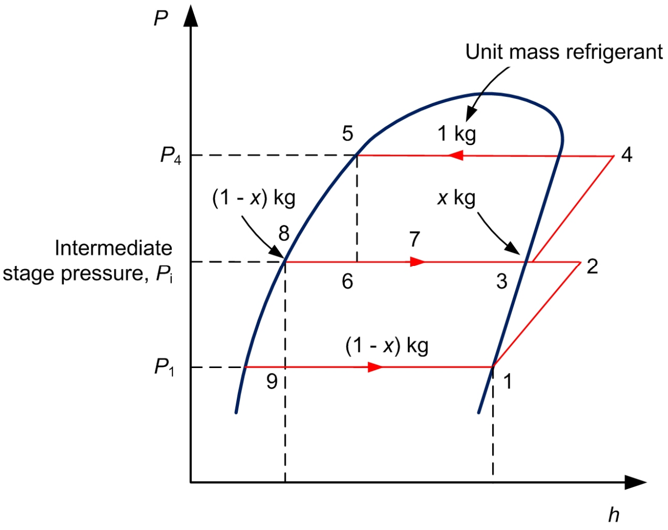


Figure 16 *p – h diagram for two-stage flash chamber vapour refrigeration system*

Figure 16 shows the $p - h$ diagram with the different pathways for unit mass of the refrigerant, for the two-stage flash chamber vapour refrigeration system, illustrated in Figure 15.

Example 11 shows the method we adopt to determine the enthalpies of the refrigerant as it circulates around the system and so determine parameters such as the COP.

Example 11

A two stage compression system operates on the cycle illustrated in Figures 15 and 16, using refrigerant 134a. The system operates between the inlet pressure of the first stage compressor at 1.327*bar* and a pressure of 8.867*bar* that leaves the second stage compressor, (the condenser operating pressure). The refrigerant leaves the condenser as a saturated liquid and is throttled to a flash chamber operating pressure of 3.496*bar*. Assuming the refrigerant leaves the evaporator as a saturated vapour and both stages of compression are isentropic, determine:

a) The fraction of refrigerant that evaporates as it is throttled to the flash chamber
b) The amount of heat taken in by the evaporator (the refrigerating effect)
c) The work done on the compressor
d) The coefficient of performance

In order to solve this problem we use the Rogers and Mayhew tables (page 15) in order to find the specific enthalpies (i.e. per unit mass) of the refrigerant at the various states indicated on Figure 15.

a) The fraction of refrigerant that evaporates, is that given by the quality of the refrigerant at *state 6*, then using Equation 2 where, $h = h_f + xh_{fg}$ or in this case $x = \dfrac{h_6 - h_f}{h_{fg}}$ where from tables with h_6 at $8.867bar$ and h_f, h_{fg} at the flash chamber pressure of $3.496bar$, we find that from the tables,

$$x = \frac{248.98 - 206.75}{194.58} = 0.217$$

b) Then heat taken in by the evaporator can be seen from Figure 15 to be $Q_{in} = (1 - x_6)(h_1 - h_9)$. Then again from tables where h_1 is a saturated vapour at $1.327bar$ and h_9 is a saturated liquid at $3.496bar$ we find that,

$$Q_{in} = (1 - 0.217)(386.44 - 206.75) = 142.49kJ/kg$$

c) The total compressor work is the work input to both stages of the compressor, which for isentropic compression is equal to $W_{in} = W_{LP} + W_{HP} = (1 - x_6)(h_2 - h_1) + (h_4 - h_3)$. The enthalpy at state 3 is determined from a specific energy balance on the mixing chamber, where the energy in is equated to the energy out (per *kg* of refrigerant). Then from the mixing point the energy out is $E_{out} = (1)(h_3)$ and $E_{in} = x_6 h_7 + (1 - x)h_2$ so that equating and from tables $h_3 = x_6 h_7 + (1 - x)h_2 = (0.217)(401.33) + (1 - 0.217)(h_2)$.

Now h_2 is found by first noting that $s_g = s_1 = s_2$ (isentropic compression) then from tables where $h_1 = 386.44kJ/kg$ at $1.327bar$ and $T_1 = -20^\circ C$ $s_1 = s_2 = 1.7408$. Now, the pressure at (2) is 3.496, therefore from this pressure line in the tables it can be seen that for the value of $s_2 = 1.7408$ a certain amount of superheat exists. So interpolating the tables the degree of superheat is given from the entropies as $\left(\dfrac{1.7562 - 1.7408}{1.7562 - 1.7238}\right) = 0.475$. Then the *increase* in enthalpy due to superheat is $= (0.475)(410.5 - 401.33) = 4.36kJ/kg$, therefore $h_2 = 401.33 + 4.36 = 405.69kJ/kg$ and from above $h_3 = x_6 h_7 + (1 - x)h_2 = (0.217)(401.33) + (1 - 0.217)(405.69) = 404.74kJ/kg$.
Now in order to find the compressor work we also need to find the enthalpy h_4, this is found in a similar manner to the way in which we found h_3. Then by interpolating tables the degree of superheat at *state 3* for enthalpy $h_3 = 404.74kJ/kg$, is $\dfrac{(410.5 - 404.74)}{10} = 0.576$ and so since entropy difference is $(17562 - 1.7238) = 0.0324$ at $p_3 = 3.4966bar$ then entropy $s_3 = 1.7238 + (0.576)(0.0324) = 1.7424kJ/kgK = s_4$.

Again, with a similar argument to above the degree of super heat at $p_4 = 8.867bar$ is, from interpolation of the entropies, $\left(\dfrac{1.747 - 1.7424}{1.747 - 1.7126}\right) = 0.134$ *so increase in enthalpy* due to superheat $= (0.134)(427.93 - 404.74) = 3.107kJ/kg$ so that $h_4 = 417.14 + 3.107 = 420.247kJ/kg$.
We now have all the enthalpies needed to determine the compressor work.
Therefore, $W_{in} = W_{LP} + W_{HP} = (1 - x_6)(h_2 - h_1) + (h_4 - h_3)$ so that:

$$W_{in} = (1 - 0.217(405.69 - 386.44) + (420.247 - 404.74) = 30.58kJ/kg$$

d) Finally, the coefficient of performance may be found using the relationship $COP = \frac{Q_{in}}{W_{comp}}$ where, as before this coefficient is the ratio of the *refrigerating effect* $\left(Q_{in}\right)$ over the *work input* (W_{comp}) and is given in this case as $COP = \frac{Q_{in}}{W_{comp}} = \frac{142.49}{30.58} = 4.66$.

Finally, you are strongly advised to work through Example 11, with the tables to hand, making sure you fully understand how the required specific enthalpies were found, including the use where necessary, of interpolation to determine the associated entropies, as well as the enthalpies.

This bring us to the end of our short introduction to vapour properties, cycles and plant, if you wish to pursue your study of vapour compression and other types of refrigeration systems, you should refer to the same standard undergraduate texts on thermodynamics, detailed at the end of Section 2.